

Paper and Pencil Math Review

Or Help for the Calculator Dependent

Fact: Being able to do some mental math is helpful even when you have a calculator – it can save you from some silly “calculator errors”.

Fact: multiplication is both commutative and associative. $(AB)C = (BC)A$

$$\text{Example: } \frac{(4)(21)}{(7)(2)} = \frac{(21)(4)}{(7)(2)} = (3)(2) = 6$$

Fact: Multiplication is distributive. $A(B + C) = AB + AC$

$$\text{Example: } (97)(40) = (100 - 3)(40) = 4000 - 120 = 3880$$

Fact: $(x^a)(x^b) = x^{a+b}$

$$\text{Example: } (10^5)(10^4) = 10^9$$

Fact: $\frac{x^a}{x^b} = x^a x^{-b} = x^{a-b}$

$$\text{Example: } \frac{6^5}{6^7} = 6^{5-7} = 6^{-2} = 1/6^2 = 1/36 \approx .03$$

Fact: $(x^a)^b = x^{ab}$

$$\text{Example: } (10^5)^3 = 10^{15}$$

$$\text{Example: } 2^6 = (2^3)^2 = 8^2 = 64$$

Fact: $\log 10^a = a$

$$\text{Example: } \log 10^5 = 5$$

Fact: $\ln e^a = a$ where $e = 2.718...$

$$\text{Example: } \ln e^{-4} = -4$$

Fact: $\ln a = 2.303 \log a$

$$\text{Example: } \ln 10^{-3} = 2.303 (\log 10^{-3}) = -3 (2.303) = -6.909$$

Fact: $p(\text{anything}) = -\log(\text{anything})$

$$\text{Example: if } K_a = 10^{-7}, pK_a = 7$$

Fact: $\log ab = \log a + \log b$

$$\text{Example: } \log (10^3)(10^5) = \log (10^3) + \log(10^5) = 3 + 5 = 8$$

Fact: $\log (a/b) = \log a - \log b$

$$\text{Example: } \log (10^5/10^3) = \log 10^5 - \log 10^3 = 5 - 3 = 2$$

Fact: $\log a^b = b \log a$

$$\text{Example: } \log (10^5)^3 = 3 \log 10^5 = (3)(5) = 15$$

Fact: Please (parantheses) Excuse (exponents) My (multiplication) Dear (division) Aunt (addition) Sally (subtraction). Don't mix up your order of operations!

Some Computational Strategies:

I. Round

Example: $(6.02 \times 10^{23})(4.1) \approx (6 \times 10^{23})(4) \approx 24 \times 10^{23}$

II. Put in scientific notation so you can deal with whole numbers and decimal places separately

Example:
$$\frac{(.06)}{(.002)(15)} = \frac{6 \times 10^{-2}}{(2 \times 10^{-3})(1.5 \times 10^1)}$$
$$= \frac{(6)}{(2)(1.5)} \frac{(10^{-2})}{(10^{-3})(10^1)} = 2(10^0) = 2$$

III. Multiply and divide in simplifying steps

Example: $\frac{(81)(42)}{(6)(9)} = \frac{(81)(42)}{(9)(6)} = (9)(7) = 63$

IV. Figure out the boundaries

Example: $\log(0.01) < \log(0.02) < \log(0.1)$ so
 $\log(0.02)$ is between -2 and -1 , but closer to -2 .
 $\log(0.02) \approx -1.7$

V. Turn division into multiplication, decimals into fractions, and division into multiplication by the inverse, as convenient.

Example: $542/0.33 = \frac{542}{1/3} = (542)(3) \approx 1600$

Example: $250(0.8) = 250(4/5) = 50(4) = 200$

VI. Break multiplication into simpler multiplication/addition problems.

Example: 15% of $\$36.00 = 10\%$ plus half again $= \$3.60 + \$1.80 = \$5.40$

Example: $(77)(31) = (80 - 3)(30 + 1) = 2400 + 80 - 90 - 3 = 2387$

VII. Know that there are fanatics who have a jillion tricks to help them – you don't need to know all the tricks but occasionally it's useful to remember a few.

Example: $77/5 = (77) \times (2/10) = 154/10 = 15.4$

Example: The digits of numbers divisible by three add up to multiples of three. $3 + 5 + 4 = 12$, so 354, 534, 345 etc. are divisible by 3

Example: The digits of numbers divisible by nine add up to multiples of nine. $5 + 4 = 9$, so 54 is divisible by 9. Six is one more than 5 so $(6)(9)=54$. 837 is divisible by 9, 839 is not.

Example: $(11)(\text{any two digit number}) = (\text{first digit in hundreds place} + \text{sum of digits in tens and hundreds place, followed by last digit in ones place})$. $(77)(11) = 70 + 14$ then $7 = 847$. $(12)(11) = 132$. $(14)(11) = 154$