abbreviations:

[E] = concentration of enzyme

$[E]_o$ = initial or original concentration of enzyme

[S] = concentration of substrate

[P] = concentration of product

[ES] = concentration of the enzyme-substrate complex

The reaction mechanism is

$$E + S \underset{k_{-1}}{\overset{k_1}{\rightleftarrows}} ES$$

$$ES \underset{k_{-2}}{\overset{k_2}{\rightleftarrows}} E + P$$

where the initial rate of the reaction

$$\text{rate} = k_2 [ES] \qquad [1].$$

The steady-state approximation assume that the [ES] is steady, i.e. a constant, thus

$$\frac{d[ES]}{dt} = rate\ of\ \ formation - rate\ of\ \ break-down = 0$$

$$= \{k_1 [E] [S] + k_{-2} [E] [P]\} - \{k_{-1} [ES] + k_2 [ES]\} = 0 \qquad [2].$$

As

$$[E] = [E]_o - [ES] \qquad [3]$$

and at the beginning of the reaction, $[P] = 0$ [4]

the substitution of [3] and [4] into [2] followed by rearrangement yields

$$[ES] = \frac{[E]_o\,[S]}{\frac{k_{-1}+k_s}{k_1} + [S]} = \frac{[E]_o\,[S]}{K_m + [S]}, \qquad [5]$$

$$\text{where}\quad K_m = \frac{k_{-1}+k_s}{k_1}.$$

Substitution of [5] into [1], yields

$$\text{Rate} = \frac{k_s\,[E]_o\,[S]}{K_m + [S]} = \frac{rate_{\max}\,[S]}{K_m + [S]}.$$

The above expression is known as the Michaelis-Menten equation, which describes the kinetics of an enzyme reacting with a single substrate.